

DW SHEET N°2
VECTOR CALCULATION

Exercise1

In repere $(O, \vec{i}, \vec{j}, \vec{k})$, we give the two vectors

$$\vec{V}_1 = \vec{i} + 2\vec{j} + 3\vec{k} \text{ et } \vec{V}_2 = 2\vec{i} - 2\vec{j} + 3\vec{k}.$$

- graphically represent these two vectors and calculate their modules.
- Calculate $\vec{S} = \vec{V}_1 + \vec{V}_2$ et $\vec{D} = \vec{V}_1 - \vec{V}_2$.
- Find the directing cosines of the direction of \vec{S} .
- Calculate the Scalar product $\vec{V}_1 \cdot \vec{V}_2$ and the angle between \vec{V}_1 et \vec{V}_2 .
- Calculate the vector product $\vec{V}_1 \wedge \vec{V}_2$.
- Determine the unit vector perpendicular to the plane (\vec{V}_1, \vec{V}_2) .

Exercise 2

Let $(\vec{i}, \vec{j}, \vec{k})$, be the unit vectors of the rectangular axes Oxyz , we consider the vectors :

$$\vec{r}_1(2,3,-1) , \vec{r}_2(3,-2,2) , \vec{r}_3(4,0,3)$$

1) Calculate their modules

2) Calculate the components and modules of the vectors

$$\vec{A} = \vec{r}_1 + \vec{r}_2 - \vec{r}_3 \text{ and } \vec{B} = \vec{r}_1 - 2\vec{r}_2$$

3) Determine the unit vector \vec{u} carried by the vector \vec{A} , what are called the components of the unit vector of A

4) Calculate $\vec{r}_1 \wedge \vec{r}_2$ and $\vec{r}_2 \cdot \vec{r}_3$, deduce the area of the triangle formed by the three vectors \vec{r}_1, \vec{r}_2 and \vec{r}_3 .

5) Calculate the products $\vec{A} \cdot (\vec{B} \wedge \vec{C})$ et $\vec{A} \wedge (\vec{B} \wedge \vec{C})$

Exercise 3

We give the three vectors:

$$\begin{cases} \vec{A} = 3\vec{i} - 4\vec{j} \\ \vec{B} = 2\vec{i} + 3\vec{k} \\ \vec{C} = \vec{i} + 2\vec{j} - 2\vec{k} \end{cases}$$

We ask :

- 1) What is the angle between \vec{A} and \vec{B}
- 2) Calculate the directing cosines of the vector \vec{C} and indicate the angles that makes \vec{C} with the three axes
- 3) The length of the projection of \vec{B} on \vec{C} .

Exercise 4

1- Determine the value of the number a for which the vectors

$$\vec{V}_1 = 2\vec{i} + a\vec{j} + \vec{k} \text{ et } \vec{V}_2 = -4\vec{i} - 2\vec{j} - 2\vec{k} \text{ be:}$$

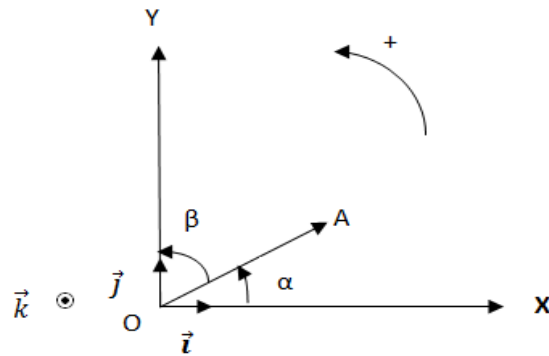
- a- perpendicular.
- b- Parallels
- c- $(\vec{V}_1 \wedge \vec{V}_2) \cdot \vec{k} = 1$

2- let C $(\alpha, 1/3, 1/2)$ be a point in space, where α is a real number. Under what condition is the (\vec{OC}) a unitary vector.

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Exercise 5

In a landmark $(O, \vec{i}, \vec{j}, \vec{k})$, direct orthonormal, we consider a vector $\vec{U} = \overrightarrow{OA}$ which makes an angle α with the axis (O, \vec{i}) and an angle β with the axis (O, \vec{j}) (Figure opposite).



- 1- the vector \vec{U} as a function of α , the module U and the unit vectors \vec{i} and \vec{j}
- 2- as a function of β , the module U and the unit vectors \vec{i} and \vec{j}
- 3- Same questions of 1 and 2 if by making a rotation in the positive direction along the axis (O, \vec{k}) of $\frac{\pi}{2}$.

Exercise 6

In an orthonormal reference (Oxyz) of unit vector $(\vec{i}, \vec{j}, \vec{k})$, we consider the vector: $\vec{V} = -2\vec{i} + 3\vec{j} + \vec{k}$ whose support passes through the point $A(3, -2, 2)$.

- 1- Calculate its moment relatively to an axis (Δ) of unit vector $\vec{U}_\Delta(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$ and which passes through the point O.
- 2- Calculate its moment with respect to the point $B(2, 1, 0)$.
- 3- Calculate its moment with respect to an axis (Δ') which passes through the point B and parallel to (Δ) .

Exercise 7 (Home work)

Let be the following vectors $\vec{U}_1 = A_1\vec{i} + A_2\vec{j} + A_3\vec{k}$ and $\vec{U}_2 = B_1\vec{i} + B_2\vec{j} + B_3\vec{k}$

- 1- Calculate scalars products $\vec{U}_1 \cdot \vec{U}_1$, $\vec{U}_1 \cdot \vec{U}_2$ and $\vec{U}_2 \cdot \vec{U}_2$
- 2- We give $\vec{V}_1 = 2\vec{i} - \vec{j} + 5\vec{k}$, $\vec{V}_2 = -3\vec{i} + 1.5\vec{j} - 7.5\vec{k}$ and $\vec{V}_3 = -5\vec{i} + 4\vec{j} + \vec{k}$
- 3- Calculate $\vec{V}_1 \cdot \vec{V}_2$ and $\vec{V}_1 \wedge \vec{V}_2$
- 4- Without making the graphic representation, what can we say about the meaning and direction of the vecteur \vec{V}_2 with respect to \vec{V}_1
- 5- Calculate the following products $\vec{V}_1 \cdot (\vec{V}_2 \wedge \vec{V}_3)$ et $\vec{V}_1 \wedge (\vec{V}_2 \wedge \vec{V}_3)$
- 6- Determine the surface of the triangle formed by the vectors \vec{V}_2 and \vec{V}_3 .

Exercise 8

The following vectors are considered:

$$\vec{V}_1 = \sin t \vec{i} - \cos t \vec{j} + 3t \vec{k}, \vec{V}_2 = \cos t \vec{i} - \sin t \vec{j}$$

Calculate $\frac{d(\vec{V}_1 \cdot \vec{V}_2)}{dt}$, $\frac{d(\vec{V}_1 \wedge \vec{V}_2)}{dt}$