Section I: Course I: Dimensions, systems of units and uncertainties

I-1 Introduction

Physics is a science that describes phenomena in a qualitative and quantitative way. It must characterize them by measurable quantities.

All these quantities have a name and are characterized by the notions of dimension and units. These concepts can be different for the same physical quantity, depending on the system used.

The dimensions of physical quantities have been the subject of the first part of this course.

By measuring a physical quantity several times, we can see that the measurements can give distinct results. It is therefore impossible to know which measurement gives the exact value of the physical quantity. This means that the instruments used are not of infinite precision and that the results obtained by the measurements present uncertainties that must be evaluated.

In order to the measurement of a physical quantity to be correct, the experimenter must pay attention to the environment in which it is carried out. The environment must be as little disturbed as possible so that the error is minimal. Therefore, there are errors in all measurements. The notion of error or uncertainty is dealt with in the last section of this part.

I-2 Dimensions

I-2-a Definition of a physical quantity

A physical quantity is a characteristic that is attributed to an object or to a phenomenon taking place in space and time. Among the set of physical quantities, some are considered independent. They are called basic quantities or fundamental quantities.

There are seven fundamental quantities.The rest, called derived quantities, are defined by means of algebraic relations linking fundamental quantities.For example, a quantity X can be given by a relation linking two quantities A, B and fixed coefficients C, α and β : $X=C.A^{\alpha}.B^{\beta}$.

I-2-b The dimensions

The dimension specifies the nature of a physical quantity.This notion is more general than the notion of unity. The dimension is represented by one or more capital letters. Each physical quantity A has a dimension, denoted:

$$\dim A=[A]$$

Exemples:

Mass, speed and energy are different quantities and therefore of different dimensions.

I-2-b-i The seven fundamental dimensions

In physics, the seven fundamental or basic quantities with symbols and dimensions are grouped in the table below.

Fundamental physical quantities.

quantities	Symbol	Dimension
Length	l,x,r	[l]=L
Mass	m	[m]=M
Time	t	[t]=T
Electrical intensity	i	[i]=I
Temperature	T	[T]=θ
Quantity of material	n	[n]= N
Light intensity	Iv	[Iv]=J

I-2-b-ii Operations on dimensions

The dimensions of physical quantities obey some basic mathematical rules:

Only quantities having the same dimension can be added together.

$a = b + c \Rightarrow [a] = [b + c] =[b] +[c]$

The dimension of a product of two quantities is the product of the dimensions of each $a = b.c \Rightarrow [a] = [b.c] =[b] .[c]$

For a greatness that can be written b^{α}

$[b^{\alpha}] = [b]^{\alpha}$. α being a real number.

The dimension of a constant is equal to 1. Exemple [2]=1.

The angles are dimensionless but have unity. [Angle]=1.

The functions sin(x), cos(x), tan(x), Ln(x), log(x) and exp(x) are dimensionless.

I-2-b-iii Dimensional equations

Dimensional equations consist in reducing the various parameters, which intervene in a physical equation or law, to the fundamental quantities of the international system of units (see paragraph I-2). These are equations that relate the dimension of a quantity G to those of the basic quantities; from which it derives. In this case the dimension of any quantity G can be in the form:

$$[G] = [\ M^a.L^b.T^c.I^d.\ \theta^e.\ N^f J^g$$

I-2-b-iv Dimension of a derived quantity

To calculate the dimension of a derived (non-fundamental) physical quantity, we need an equation, which relates it to the fundamental quantities.This is also possible using the unit of quantity.

Exemples :

The speed:

from the equation: V = (dx/dt) ⇒ $[V] = L.T^{-1}$.

From the unit: unit of V : m/s ⇒ $[V] = L.T^{-1}$.

The acceleration:

from the equation: a = (dV/dt) ⇒ $[a] = L.T^{-2}$.

From the unit: unit of a: m/s^2 ⇒ $[a] = L.T^{-2}$.

In the table below, we give some examples of derived quantities and their dimensions.

Dimensions of some derived quantities.

quantities	Equation	Dimension
Angular velocity, pulsation:	α/t	T^{-1}
Angular acceleration	ω/t	T^{-2}
Frequency:f	1/T	T^{-1}
Force:F	m.a	$M.L.T^{-2}$
Moment of inertia: J	$m.l^2$	$M.L^2$
Pressure:pr	F/S	$M.L^{-1}.T^{-2}$
Work:W	F.d	$M.L^2.T^{-2}$
Power:P	W/t	$M.L^2.T^{-3}$
Electric charge:Q	i.t	I.T
Potential:V	P/i	$M.L^2.T^{-3}.I^{-1}$
Electric field: E	V/l	$M.L.T^{-3}.I^{-1}$
Capacity:C	Q/V	$M^{-1}.L^{-2}.T^4.I^2$
Resistance:R	V/i	$M.L^2.T^{-3}.I^{-2}$
Magnetic field:B	F/q.V	$M.T^{-2}.I^{-1}$
Inductance:L	V/(di/dt)	$M.L^2.T^{-2}.I^{-2}$

I-2-b-V Dimensional analysis

Dimensional analysis allows:

1- To determine the dimension and the unit of a derived physical quantity, according to the dimensions and the units of the fundamental quantities.

2- To carry out changes of units.

3- To check the homogeneity of the formulas: $A = B \Rightarrow [A] = [B]$.

The analysis of the homogeneity of an equation is a tool that allows the detection of errors in a law. A non-homogeneous equation is necessarily false.

I-3 Systems of units

I-3-a Measurement of a quantity

To measure a physical quantity X, it suffices to compare it with the same physical quantity of different value, taken as a reference. The latter constitutes what is called a standard.

The value of a physical quantity X is generally in the form of the product of its unity by a real number. For certain particular quantities, Several different units can be used. For example, in the case of the speed of an object, V can be written in the form:

V =25m/s ou bien V =90Km/h.

I-3-b International System of units

It is possible to deduce a correct unit of a physical quantity, from this dimension.

To create a system of units, such as the international system of unity (SI), it is necessary, first of all, to establish a system of quantities and a series of equations defining the relationships between these quantities. This is essential because the equations connecting the quantities together determine those connecting the units together.

In 1960, the 11th General Conference on Weights and Measures established the international system of units (SI), also called MKSA (Meter - Kilogram, second, Ampere).The conference also set rules for prefixes, derived units and others. The SI is based on seven well-defined fundamental units, considered from the dimensional point of view.

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Fundamental units of the system (SI).

Quantities	Name	Symbol
Length	meter	m
Mass	Kilogram	Kg
Time	Second	s
Electrical intensity	Ampere	A
Temperature	Kelvin	K
Quantity of material	Mole	mol
Light intensity	Candela	Cd

I-3-c The C.G.S system

In addition to the MKSA system, there is another system of units, so-called CGS. The latter was created by the "British Association" in 1873. This system uses as fundamental units: the centimeter for length, the gram for mass and the second for time.The units of length and mass of the CGS system are decimal submultiples of the units corresponding to the MKSA system. The formulas defining the derived units are the same for both systems.

I-3-d Derived units

The derived units are formed from the basic (fundamental) units and this thanks to the mathematical relationships that connect them.

Exemple:

The current density in (A / m^2), the volume in (m^3) and the density in (Kg / m^3). Some units were given special namesand symbols. Some examples, in both systems of units, are provided in Tables (4) and (5).

Derived units for some physical quantities, in the MKSA system.

Derived quantity	Name of the unit	Symbol	Unit in the MKSA system
Force	Newton	N	$M.Kg.s^{-2}$
Pressure	Pascal	Pa	$m^{-1}.Kg.s^{-2}$
Energy	Joule	J	$m^2.Kg.s^{-2}$
Power	Watt	W	$M^2.Kg.s^{-3}$
Frequency	Hertz	Hz	s^{-1}
Electric charge	Coulomb	C	s.A
Potential difference	Volt	V	$m^2.Kg.s^{-3}.A^{-1}$
Resistance	Ohm	Ω	$m^2.Kg.s^{-3}.A^{-2}$
Magnetic induction	Tesla	T	$Kg.s^{-2}.A^{-1}$
Magnetic inductance flux	Weber	Wb	$m^2.Kg.s^{-2}.A^{-1}$

Derived units for some physical quantities, in the system. CGS.

quantities	Name of the unit	Symbol	Equivalent in the CGS system
Force	dyne	dyn	$1dyn=10^{-5}N$
Pressure	Baryes	Bary	$1Bary=10^{-1}$ Pa
Energy	erg	erg	$1erg=10^{-7}J$
acceleration	gal	Gal	$1Gal=10^{-2}m.s^{-2}$
Magnetic field	Oersted	Oe	$1Oe = 10^3/4\pi\ A.m^{-1}$
Magnetic induction	Gauss	G	$1G=10^{-4}T$
Dynamic viscosity	poise	P	1P=0.1Pa.s
Kinematic viscosity	stokes	St	$1\ St=10^{-4}m^2.s^{-1}$

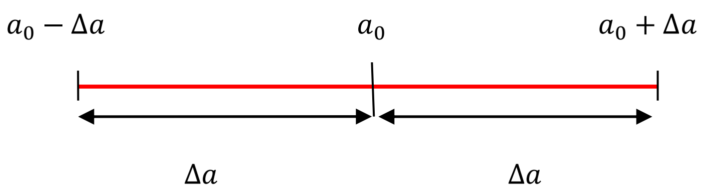
I-4 The uncertainties

I-4-a Notion of error or uncertainty

The Error or uncertainty is the deviation between the exact value and the value obtained by the measurement. The exact value of the physical quantity is often inaccessible. To get closer to this value, it is necessary to estimate this error well.We distinguish two types of error.

I-4-b The absolute uncertainty

The absolute uncertainty is the estimate of the error that the experimenter makes. This is the possible deviation between the value obtained by the measurement and the exact value. The measurement and its uncertainty constitute a domain of possible values within which the exact value is located. Let be a physical quantity a, whose exact value is a0. The writing: $a = a_0 \pm \Delta a$ is to say that the value of a is included in the interval:: $[a_0 - \Delta a , a_0 + \Delta a]$.



The absolute uncertainty is a positive real number, it is expressed in the units of the measured magnitude. Sometimes the absolute error is attributed to the accuracy of the instrument used during the measurement.

Exemple:

The measurements of the dimensions of a room give the following values: Length: L = (10.2 ± 0.1) m. Width: l = (7.70 ± 0.08) m. Height : H = (3.17±0.04) m

Calculate and give the results with their absolute uncertainties:

a- The surface S of the floor

b- The volume V of the room.

Solution:

a- S=L.l= 78.54 m². puisque $\frac{\Delta S}{S} = \frac{\Delta L}{L} + \frac{\Delta l}{l} \Rightarrow \Delta S = S.\left(\frac{\Delta L}{L} + \frac{\Delta l}{l}\right)$, ΔS = 1.59 m².

S= (78.54 ± 1.59) m^2.

b- V=L.l.h= 248.97 m^3. Puisque $\frac{\Delta V}{V} = \frac{\Delta L}{L} + \frac{\Delta l}{l} + \frac{\Delta h}{h} \Rightarrow \Delta V = V.\left(\frac{\Delta L}{L} + \frac{\Delta l}{l} + \frac{\Delta h}{h}\right)$, ΔV=8.17 m^3.

V= (248.97 ± 8.17) m^3

I-4-c The relative uncertainty

For a physical quantity a, the quality or accuracy of the measurement is given by the relative uncertainty, which is defined as the ratio between the absolute uncertainty and the measured value, called a/a. This ratio is usually expressed as a percentage.

Example :

Uncertaintie of 1 mm on a measurement of 5 cm corresponds to a relative accuracy of $\Delta x/x = 10^{-3}$ m / 5.10^{-2} = 0.02 (2%).

Cette même incertitude sur une mesure de 5 m correspondrait à une précision, excellente, de 10^{-3} m / s = 0.0002 (0,02%).

I-4-d Operations on uncertainties

The uncertainties also obey certain basic mathematical operations.

Case of a sum or a difference: If a physical quantity C is the resultant of two quantities B and C.

if C = A + B, so ΔC=ΔA+ΔB

if C = A - B, alors ΔC=ΔA+ΔB

The relative Uncertainty on C is :ΔC/C

Case of a product or a ratio: If a physical quantity G is the result of a product or a ratio of quantities, for example A, B et C:

$G = A.B.C$: $(\Delta G/G) = (\Delta A/A) + (\Delta B/B) + (\Delta C/C)$

$G = A.B / C$:$(\Delta G/G) = (\Delta A/A) + (\Delta B/B) + (\Delta C/C)$

The previous result can be obtained using the logarithm function. Let be the slightly more general case of the quantity G, which is written: $G=(K.A^{\alpha}.B^{\beta})/C^{\gamma}$

A,B and C are physical quantities that are measured and K is a constant, the calculation of the uncertainty, in this case, the relative uncertainty on the result is obtained according to the following steps:

Let's apply the logarithm function to the two members the expression of G:

$\text{Log } G= \log [(K.A^{\alpha}.B^{\beta})/C^{\gamma}] \Rightarrow \text{Log } G = \text{Log } K +\alpha \text{ Log } A +\beta \text{ Log } B- \gamma \text{ Log } C$

1- The differential of the expression obtained is: $dG/G=\alpha\, (dA/A)+\beta\, (dB/B)-\gamma\, (dC/C)$

1– The differential elements (dG, dA, dB, dC) are infinitesimal quantities or very small deviations (variations) in the quantities G, A, B et C. If these differences become significant, these elements become uncertainties (ΔG, ΔA, ΔB, ΔC). Which gives:

$\Delta G/G = \alpha\, (\Delta A/A)+\beta\, (\Delta B/B) +\gamma\, (\Delta C/C)$

Note that in the above equation, the sign (-) has been replaced by the sign (+). Which is explained by the fact that errors accumulate or add up. Finally, we will have :

$$G_{corrigée} = G_{mesurée} \pm \Delta G.$$