Fiche de TD n° 03

Calcul vectoriel

Exercice 1 : $\vec{V_1} = 3\vec{i} - 4\vec{j} + 4\vec{k}$, $\vec{V_2} = 2\vec{i} + 3\vec{j} - 4\vec{k}$.
$\vec{V_3} = 5\vec{i} - \vec{j} + 3\vec{k}$.

a/ les modules : $V_1 = 6,4$, $V_2 = 5,38$, $V_3 = 5,91$.

b/ Le vecteur unitaire porté par $\vec{C} = \vec{V_1} + \vec{V_3}$.

$\vec{C} = 8\vec{i} - 5\vec{j} + 7\vec{k}$, $C = 11,74$.

$\vec{U_C} = \frac{\vec{C}}{C} \Rightarrow \vec{U_C} = \frac{8}{11,74}\vec{i} - \frac{5}{11,74}\vec{j} + \frac{7}{11,74}\vec{k}$.

c/ Le produit scalaire

$\vec{V_1} \cdot \vec{V_3} = x_1 x_3 + y_1 y_3 + z_1 z_3 = 15 + 4 + 12 \Rightarrow \vec{V_1} \cdot \vec{V_3} = 31$

$\vec{V_1} \cdot \vec{V_3} = V_1 \cdot V_3 \cdot \cos\alpha \Rightarrow \cos\alpha = \frac{\vec{V_1} \vec{V_3}}{V_1 V_3}$

$\Rightarrow \cos\alpha = \frac{31}{6,40 \cdot 5,91} = 0,82 \Rightarrow \alpha = 35$

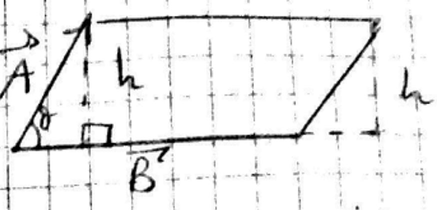
d/ Le produit vectoriel $\vec{V_2} \wedge \vec{V_3} = \vec{W}$

$$\vec{W} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & -4 \\ 5 & -1 & 3 \end{vmatrix} = 5\vec{i} - 26\vec{j} - 17\vec{k} .$$

Ex02 :

$$S = B.h = B.A\sin\theta$$

$$\rightarrow S = |\vec{A}\wedge\vec{B}| = A.B\sin\theta .$$



$$\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$$

$$\vec{B} = B_x\vec{i} + B_y\vec{j} + B_z\vec{k} .$$

$$|\vec{A}+\vec{B}| = \sqrt{(A_x+B_x)^2 + (A_y+B_y)^2 + (A_z+B_z)^2}$$

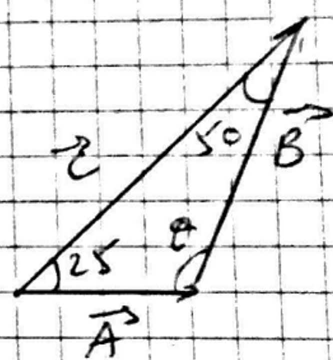
$$|\vec{A}-\vec{B}| = \sqrt{(A_x-B_x)^2 + (A_y-B_y)^2 + (A_z-B_z)^2}$$

$$|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}| \rightarrow (A_x+B_x)^2 + (A_y+B_y)^2 + (A_z+B_z)^2 - (A_x-B_x)^2$$
$$-(A_y-B_y)^2 - (A_z-B_z)^2 = 0$$

$$\rightarrow A_xB_x + A_yB_y + A_zB_z = 0 \rightarrow \vec{A}\perp\vec{B} . \ (\text{car } \vec{A}\cdot\vec{B} = 0)$$

Ex3 :

$$|\vec{C}| = 30 , (\vec{A},\vec{C}) = 25° , (\vec{B},\vec{C}) = 50°$$



$$|\vec{C}| = |\vec{A}+\vec{B}| = 30 \ ; \ \theta = 180 - (25+50)$$

$$\theta = 105° .$$

En utilisant la loi des sinus :

$$\frac{A}{\sin 50} = \frac{B}{\sin 25} = \frac{C}{\sin 105} = \frac{30}{\sin 105}$$

$$\rightarrow A = 23{,}8 \ , \quad B = 13{,}1 .$$

B

Ex4 :

$$\vec{A} = \begin{pmatrix} 1 \\ \alpha \\ \beta \end{pmatrix}, \vec{B} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$$

$$\vec{B} \parallel \vec{A} \Rightarrow \vec{B} = \lambda \vec{A} \quad (\lambda = cte)$$

$$\Rightarrow \frac{\vec{B}}{\lambda} = \vec{A} \Rightarrow \begin{pmatrix} \frac{2}{\lambda} \\ -\frac{3}{\lambda} \\ \frac{4}{\lambda} \end{pmatrix} = \begin{pmatrix} 1 \\ \alpha \\ \beta \end{pmatrix}$$

$$\Rightarrow \begin{cases} \frac{2}{\lambda} = 1 \\ -\frac{3}{\lambda} = \alpha \\ \frac{4}{\lambda} = \beta \end{cases} \Rightarrow \begin{cases} \lambda = 2 \\ \alpha = -1,5 \\ \beta = 2 \end{cases} \Rightarrow \vec{B} \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}, \vec{A} \begin{pmatrix} 1 \\ -1,5 \\ 2 \end{pmatrix}$$