Section IV: Dynamics of the material point

IV-1 Introduction

Unlike kinematics, treated in the previous part, dynamics is the part of mechanics that allows movements to be treated, taking into account their causes and which are the forces.

This part comprises a certain number of important laws or principles making it possible to link the forces and the kinematic elements, in a movement of an object.

In this section, we will describe different types of forces as well as the notion of angular momentum. The latter being useful for the study of the particular cases of circular and rotational movements.

IV-2 Fundamental laws of dynamics

Newton's laws are three in number.

IV-2-a Newton's First law: Principle of inertia

Satatement of Newton's first law

"In the absence of external forces, an object at rest remains at rest and an object in motion continues to move in a uniform rectilinear manner".

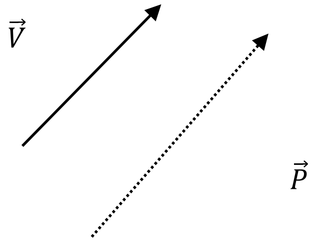
In the absence of external forces or if the vector sum of applied forces is zero, the system or the object is said to be isolated.

IV-2-b Newton's second law: Fundamental principle of dynamics

Two concepts are introduced into this law. The mass (m) of the body and its momentum (P).Mass is the physical quantity that measures the inertia of a body. In other words, if the mass of an object is greater, the more difficult it will be to impose on it: acceleration, deceleration (slowing down) or a change of direction.

We know that the movement of a body is described by the kinematic vectors: position, speed and acceleration.But this information is insufficient to describe the condition of the body. For this, it is necessary to introduce additional quantities. Among these quantities, the quantity of the movement. This quantity combines the speed of the body with its mass.

- ✓ The momentum $\vec{P}$ of an object is defined as being the product of its mass by its velocity vector: $\overrightarrow{P} = m\vec{V}$.
- ✓ The momentum is a vector quantity that has the same direction as the velocity



Note: A free body moves with a constant amount of movement.

Satatement of Newton's second law

"In a Galilean Reference , the vector sum of the forces exerted on a material point is equal to the product of the acceleration vector and its mass"

$$\sum \vec{F}_{ext} = m.\vec{a}$$

This law makes it possible to relate the kinematics of the material point to the causes of its movement. In the general case, the resultant of the forces exerted on an object is equal to the derivative, with respect to time, of the momentum vector:

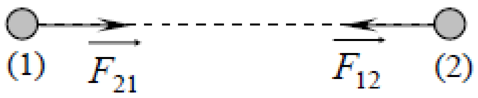
$$\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt}$$

In the particular case of a constant mass:

$\sum \vec{F}_{ext} = \frac{d(m\vec{V})}{dt} = \frac{dm}{dt}\vec{V} + m\frac{d\vec{V}}{dt}$ if m is constant, $\frac{dm}{dt} = 0 \Rightarrow \sum \vec{F}_{ext} = m\frac{d\vec{V}}{dt} = m\vec{a}$

IV-2-c Newton's Third Law: action and reaction

If an object (1) exerts a force $\vec{F}_{1\to2}$ on another object (2), the latter in return exerts a force $\vec{F}_{2\to1}$, of the same intensity but in the opposite direction $\vec{F}_{1/2} = -\vec{F}_{2/1}$



If the system is isolated $\Rightarrow$ $\sum \vec{F}_{ext} = \vec{0} \Rightarrow \frac{d\vec{P}_1}{dt} + \frac{d\vec{P}_2}{dt} = \vec{0} \Rightarrow \vec{F}_{1/2} + \vec{F}_{2/1} = \vec{0}$

These forces are carried by the same straight line.

IV-3 Examples of forces

we call a force any action from the outside on a system and which would imply a change in the state of rest.

IV-3- a The weight

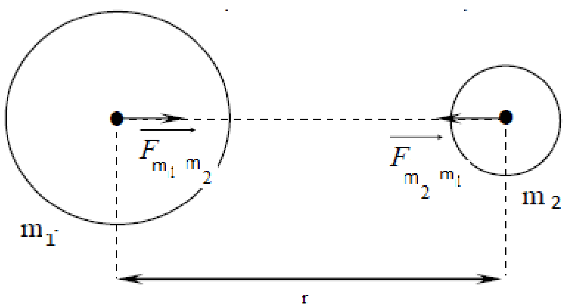
The weight of a body is the force exerted by the earth on an immobile body. This force is called the force of gravity.$\vec{F} = \vec{P} = m\vec{g}$, where $\vec{g}$ is the acceleration vector of gravity. The value of g is 9.8m/s^2.

IV-3-b Universal gravitation

If M1 and M2 are two objects of masses m1 and m2, separated by a distance r, they are interacting. The force of attraction that appears between them is:

$$\vec{F} = G.\frac{m_1.m_2}{r^2}$$

G being the gravitational constant equal to 6.673 10^{-11} N.m²/Kg². A good example is that of the moon-earth interaction.



The force exerted by the earth on an object is written: $F_g = m.g(r) = m.\left(\frac{GM_T}{r^2}\right)$

The gravitational force in the vicinity of the earth is the weight: $F_g = m.g(r) = m.\left(\frac{GM_T}{r^2}\right)$

où $g(r) = m.\left(\frac{GM_T}{r^2}\right)$

The gravitational force in the vicinity of the earth is: $m.\frac{GM_T}{{R_T}^2} = mg_0$, $g_0 = \frac{GM_T}{{R_T}^2}$

M_T : is the mass of the earth equal to 5.98 10^{34} Kg.

R_T : the radius of the earth equal to $6.37.10^6$ m.

g_0 : the gravity field equal to 9.8 m/s^2.

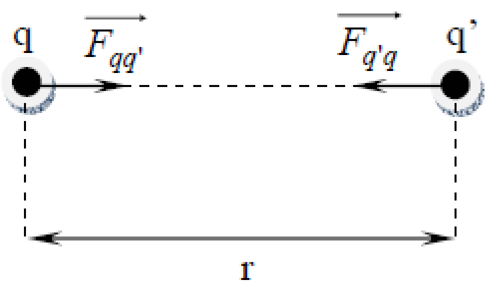
If the body is at a height Z from the earth's surface, then:

$$F = \frac{m.M}{(R+Z)^2} \qquad g = \frac{GM}{(R+Z)^2} = \frac{GM}{R^2} \cdot \frac{R^2}{(R+Z)^2} = g_0 \frac{R^2}{(R+Z)^2}$$

IV-3-c Coulomb's law in electrostatics

The Coulomb interaction is the equivalent of the gravitational interaction for electric charges. Two charges q1 and q2, of opposite signs, attract each other. The modulus of the force of attraction is given by:

$$\vec{F}_{q'q} = \frac{1}{4\pi\varepsilon_0}\frac{q'q}{r^2} = k\frac{q'q}{r^2}$$



k is a constant that depends on the medium in which the two charges are located. k= $\frac{1}{4\pi\varepsilon_0} = 9.10^9$ USI

IV-3-d Electromagnetic interaction: Lorentz force

The electromagnetic field exerts a force on particles possessing an electric charge q, in non-zero motion X. The force that an electric charge placed in an electric field E $^{\rightarrow}$ and magnetic B $^{\rightarrow}$ is called the electromagnetic force or Lorentz force:

$$\vec{F} = q.(\vec{E} + \vec{V}\Lambda\vec{B}).$$

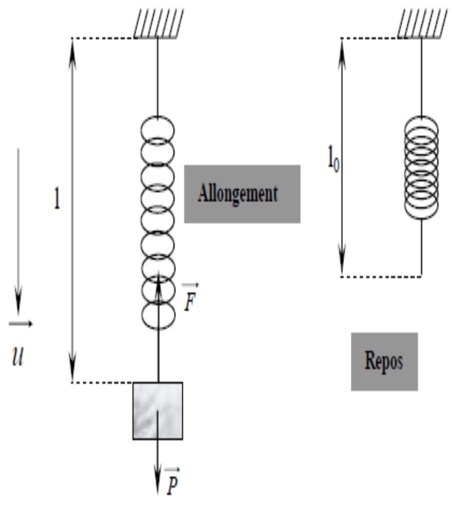
The vectors $\vec{E}$ and $\vec{B}$ are taken at the point where the electric charge is located. $\vec{V}$ represents the speed of the electric charge in the study landmark.

IV-3-e The return force of a spring.

A mass (m) hooked to a spring of no-load length L0 undergoes a return force $\vec{F}$, once the spring is elongated, with a length L.

$\vec{F}$ is given by:

$$\vec{F} = k.\Delta l.\vec{u}$$

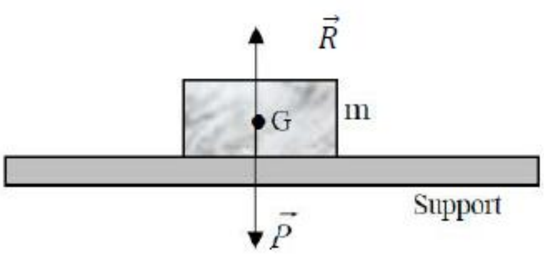


k: is the stiffness constant, characteristic of the spring, which is expressed in$N.m^{-1}$.where ΔL is the differenceL-L_0.

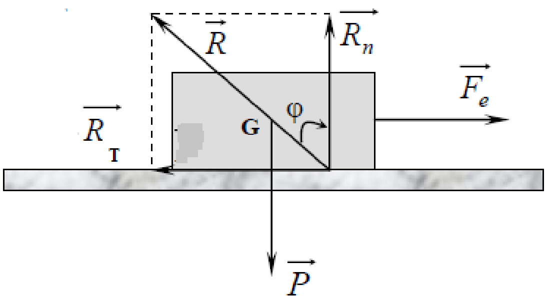
IV-3- Contact forces

IV-3-f-i Reaction of the support

An object of weight (P), placed on a horizontal support undergoes the reaction of the support.La direction of this reaction is orthogonal to the surface of the support at the level of the contact.



IV-3-f-ii Frictional forces



Friction is the action of a rigid surface on a solid. This action opposes the movement relative to the surface. For example, when pushing an object on a table with a speed and after dropping it, the object slows down and stops.The loss of momentum shows that a force R ⃗opposes the movement. This force is called the frictional force.

The tangential reaction ratio $\vec{R}_T$ and normal R ⃗Ndefines what is called the coefficient of friction. µ. $\mu = \frac{R_T}{R_N}$.

If the body is at rest, the static friction coefficient is defined.$\mu_s = \frac{R_T}{R_N}$.

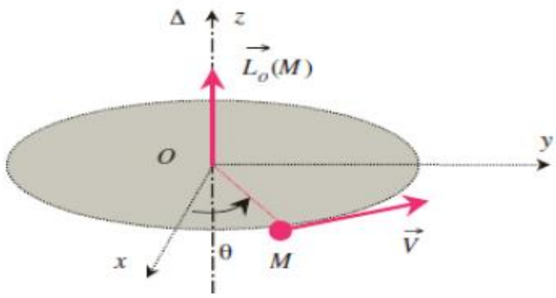
If the body is in motion, the kinetic friction coefficient is defined.: $\mu_c = \frac{R_T}{R_N}$.

IV-4 Angular momentum

In several cases, as for example in the case of rotational movements, it is more convenient to use the angular momentum theorem instead of Newton's second law. In what follows, we will consider the movement of a material point M, of mass m in a reference frame O and relative to a fixed point O of the landmark.

IV-4-at angular momentum of a material point

Let be a material point M of mass m and in motion with a speed V, with respect to a center O and in a reference frame V. The angular momentum of M, with respect to O is defined by: $\vec{L}_0(M)_R = \overrightarrow{OM} \Lambda m\vec{V}$



The kinetics of the material point M with respect to an axis (X) are also defined as follows:

$\vec{L}_\Delta(M)_R = (\overrightarrow{OM} \Lambda m\vec{V})\,.\vec{u}$ $\vec{u}$ being the unit vector carried by the axis (Δ).

IV-4-b Angular momentum theorem

Let an object of mass (m) be moving at a speed $\vec{V}$, in a Galilean repository $\Re$.

IV-4-b-i Kinetic moment theorem with respect to a fixed point O

« In a Galilean reference frame$\mathcal{R}$, the dynamic moment $\frac{\mathrm{d}}{\mathrm{dt}}\vec{\mathrm{L}}_{/\mathrm{O}}$ of a material point M, with respect to a fixed point Odu reference frame, is equal to the moment of the resultant of the external forces exerted on M

$$\frac{d}{dt}\vec{L}_{/O} = \overrightarrow{OM}\Lambda\, m\,\vec{a} = \overrightarrow{OM}\Lambda\sum\vec{F} = \mathcal{M}_{\mathrm{O}}(\vec{\mathrm{F}})$$

Indeed, if a $^{\rightarrow}$ is the acceleration of the movement:

$$\frac{d}{dt}\vec{L}_0 = \frac{d(\overrightarrow{OM}\Lambda m\vec{V})}{dt} = \frac{d\overrightarrow{OM}}{dt}\Lambda m\vec{V} + \overrightarrow{OM}\Lambda m\frac{d\vec{V}}{dt} \qquad \frac{d}{dt}\vec{L}_0 = \vec{V}\Lambda m\vec{V} + \overrightarrow{OM}\Lambda m\frac{d\vec{V}}{dt} = \overrightarrow{OM}\Lambda m\vec{a}$$

Knowing that (V) $^{\rightarrow}\Lambda m\vec{V} = \vec{0}$ and using Newton's second law, we get:

$$\frac{d}{dt}\vec{L}_{/O} = \overrightarrow{OM}\Lambda m\vec{a} = \overrightarrow{OM}\Lambda\sum\vec{F}$$

IV-4-b-ii Kinetic moment theorem with respect to an axis (Δ)

With respect to an axis (Δ), the dynamic moment$\frac{\mathrm{d}}{\mathrm{dt}}\vec{\mathrm{L}}_{/(\Delta)}$ is given by:

$$\frac{d}{dt}\vec{L}_{/\Delta} = \overrightarrow{OM}\Lambda\, m\,\vec{a} = \overrightarrow{OM}\Lambda\sum\vec{F} = \mathcal{M}_{\Delta}(\vec{\mathrm{F}})$$

« $\frac{d}{dt}\vec{L}_{/(\Delta)}$ *is the moment, relative to the axis (Δ),of the resultant of the external forces applied to the body in motion "*