(06)

2) $y'' + 2y' = \cos(2x) \quad y(0) = y'(0) = 0$

$\mathcal{L}(y'') + 2\mathcal{L}(y') = \mathcal{L}(\cos(2x))$

$s^2\mathcal{L}(y) - \cancel{s}\, \underbrace{y'(0)}_{0} - s\underbrace{y(0)}_{0} + 2s\mathcal{L}(y) - 2\underbrace{y(0)}_{0} = \frac{s}{s^2+4}$

$(s^2+2s)\mathcal{L}(y) = \frac{s}{s^2+4} \Rightarrow \mathcal{L}(y) = \frac{s}{(s^2+2s)(s^2+4)}$

$= \frac{\cancel{s}}{\cancel{s}(s+2)(s^2+4)}$

$= \frac{1}{(s+2)(s^2+4)}$

$\Rightarrow \mathcal{L}(y) = \frac{a}{s+2} + \frac{bs+c}{s^2+4}$

$= \frac{1}{8}\left(\frac{1}{s+2}\right) - \frac{1}{8}\frac{s}{s^2+4} + \frac{1}{4}\frac{1}{s^2+4}$

$\Rightarrow \mathcal{L}^{-1}\mathcal{L}(y) = \frac{1}{8}e^{-2x} - \frac{1}{8}\cos 2x + \frac{1}{4} \times \mathcal{L}^{-1}\left(\frac{2}{2(s^2+4)}\right)$

$y(x) = \frac{1}{8}e^{-2x} - \frac{1}{8}\cos 2x + \frac{1}{8} \cdot \sin 2x$

exo n° 5:

1) $y^{(4)} - y = \sinh 2x$

$\Rightarrow \mathcal{L}(y^{(4)} - y) = \mathcal{L}(\sinh 2x) = \frac{2}{s^2-4}$